

# Spiking Neuron Networks

## A Survey

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# Overview

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- Motivation
- Biology
- SNN Models
- Temporal Coding
- ESN's and LSM's
- Computational Power of SNNs
- Training/Learning with SNNs
- Software/Hardware Implementation
- Applications
- Discussion

# Neural Network Classification

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## ○ 1<sup>st</sup> Generation:

- Perceptrons, Hopfield Networks, MLP with threshold units

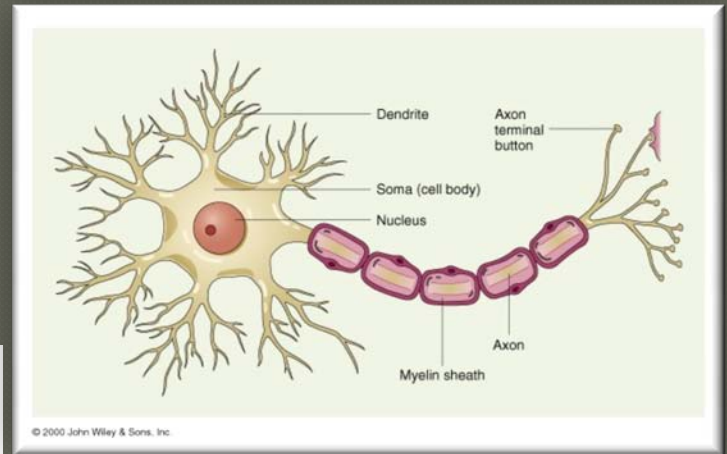
## ○ 2<sup>nd</sup> Generation:

- Networks with non-linear activation units and real-valued, continuous set of output units

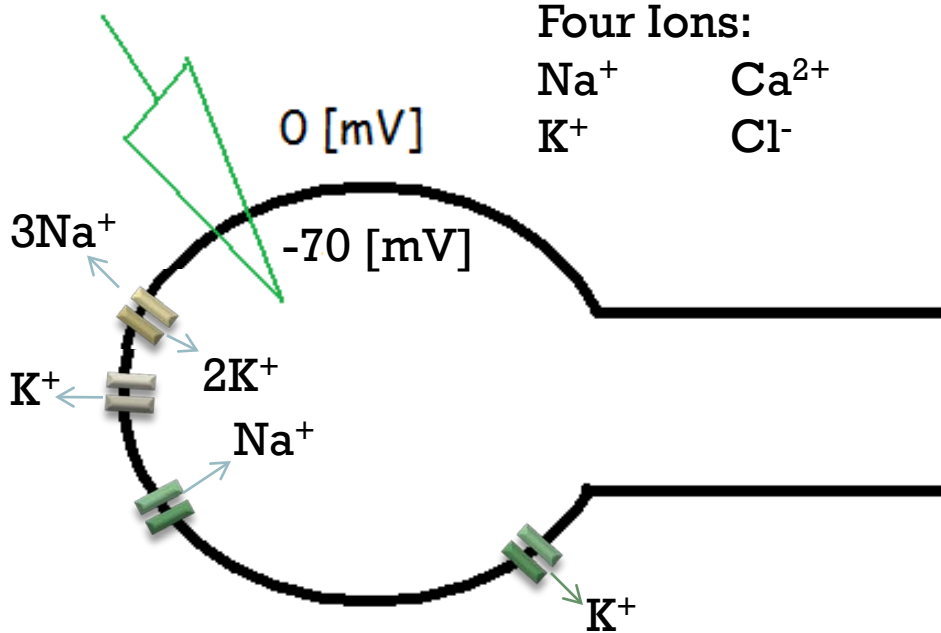
## ○ 3<sup>rd</sup> Generation:

- Spiking neuron networks, using firing times of neurons for information encoding

# The Biological Neuron

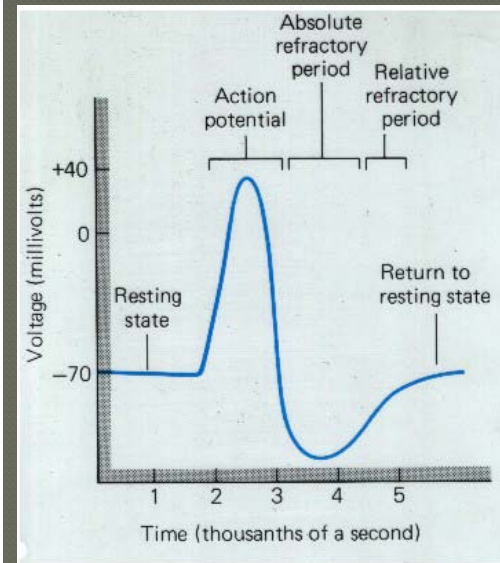


Membrane Electrode



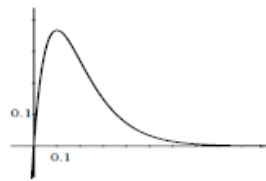
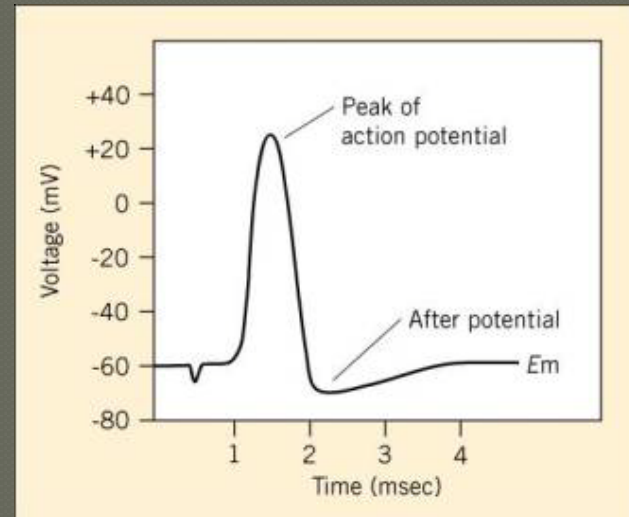
Four Ions:

$\text{Na}^+$        $\text{Ca}^{2+}$   
 $\text{K}^+$          $\text{Cl}^-$

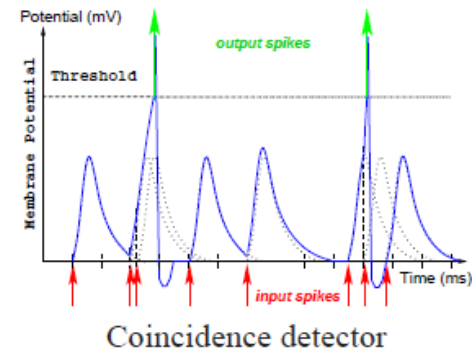
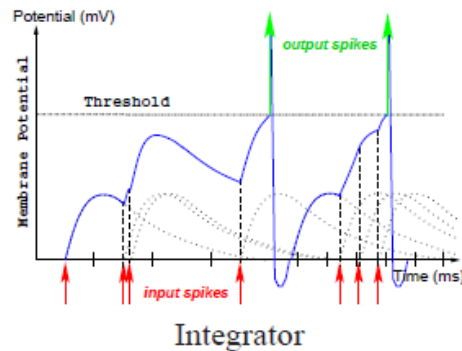


# Traditional Spike Representation

- Alpha Function
- Integrator
- Coincidence Detector



Example of  $\alpha$ -function:  
 $f(t) = \frac{t}{0.1} * \exp(-\frac{t}{0.1})$

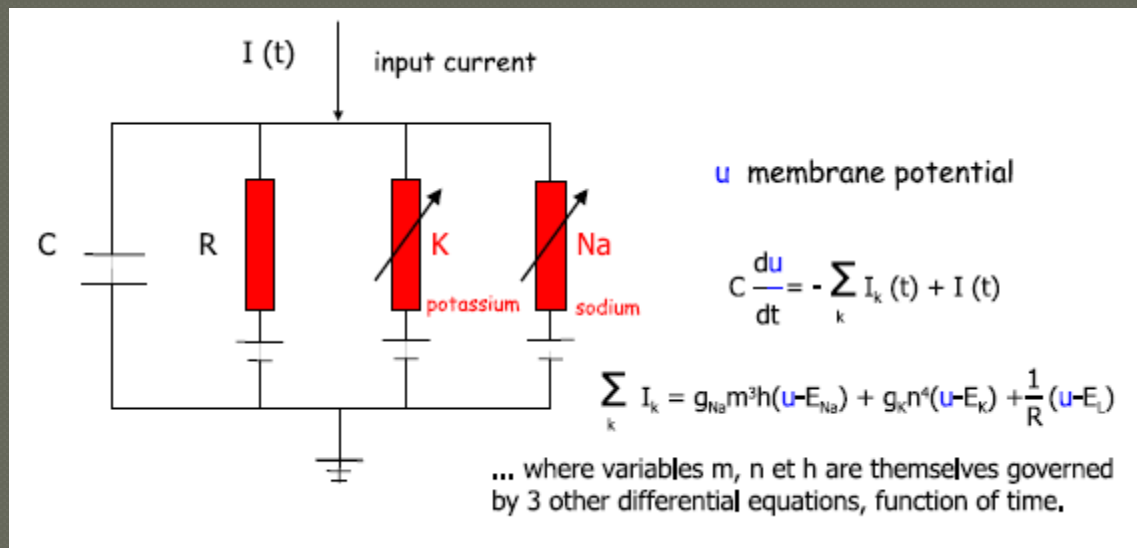


# Hodgkin-Huxley Model

## Models membrane potential

- Conductance-based
- Defined in 1952 (Note: Na-K Pump disc. in 1957)

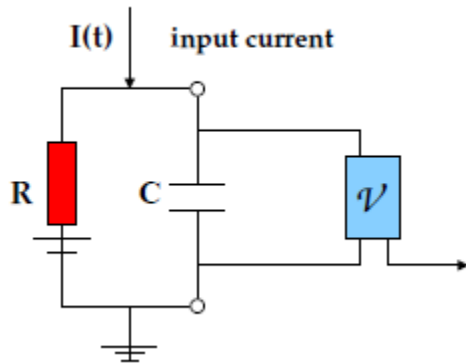
$$C \frac{du}{dt} = -g_{Na} m^3 h (u - E_{Na}) - g_K n^4 (u - E_K) - g_L (u - E_L) + I(t)$$



# Leaky Integrate & Fire Model

- Considers spike as event
- Ions leak out, requiring time constant,  $\tau$

$$\tau_m \frac{du}{dt} = u_{rest} - u(t) + RI(t)$$



**u** membrane potential

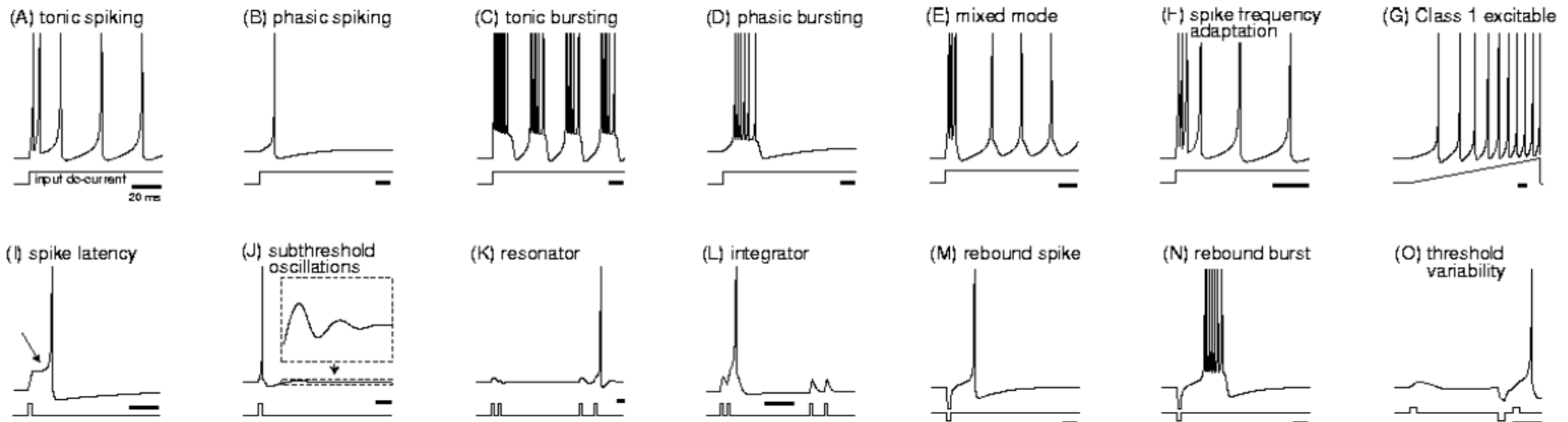
$$C \frac{du}{dt} = -\frac{1}{R}u(t) + I(t)$$

spike emission time  $t^{(f)}$  is defined by

$$u(t^{(f)}) = \vartheta \quad \text{with} \quad u'(t^{(f)}) > 0$$

# Izhikevich's Firing Behaviors

- 20 Possible Neuron Firing Behaviors
- LIF can only accommodate 3 (A, G, & L)





# Izhikevich Neuron Model

## ○ Two variables

- Voltage Potential ( $v$ )
- Membrane Recovery (activation of K currents and inactivation of Na currents) ( $u$ )
- $W$  is the weighted input(s),  $a$ ,  $b$ ,  $c$  &  $d$  are abstract parameters of the model

$$\frac{du}{dt} = a(bv - u)$$

$$\frac{dv}{dt} = .04v^2 + 5v + 140 - u + W$$

## ○ When ( $v > \text{threshold}$ ), $v$ and $u$ are reset:

$$v \rightarrow c$$

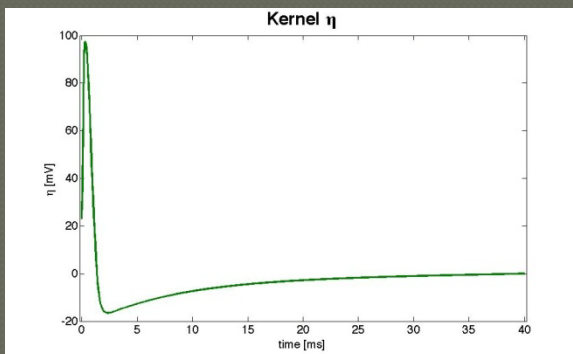
$$u \rightarrow u + d$$

# Spike Response Model

- Adds a refractory period

$$u_j(t) = \sum_{t_j^{(f)} \in \mathcal{F}_j} \underbrace{\eta_j(t - t_j^{(f)})}_{\mathbf{s}} + \sum_{i \in \Gamma_j} \sum_{t_i^{(f)} \in \mathcal{F}_i} w_{ij} \underbrace{\epsilon_{ij}(t - t_i^{(f)})}_{\mathbf{s}} + \underbrace{\int_0^\infty \kappa_j(r) I(t - r) dr}_{\text{if external input current}} \quad (4)$$

Spike & Spike Reset



Weighted Sum of Inputs

$$\eta_j(s) = -\eta_0 \exp\left(-\frac{s - \delta^{abs}}{\tau}\right) \mathcal{H}(s - \delta^{abs}) - K \mathcal{H}(s) \mathcal{H}(\delta^{abs} - s)$$

$$\eta_j(s) = -\vartheta \exp\left(-\frac{s}{\tau}\right) \mathcal{H}(s)$$

$$\epsilon_{ij}(s) = \frac{s - d_{ij}^{ax}}{\tau_s} \exp\left(-\frac{s - d_{ij}^{ax}}{\tau_s}\right)$$

$$\epsilon_{ij}(s) = \left[ \exp\left(-\frac{s - d_{ij}^{ax}}{\tau_m}\right) - \exp\left(-\frac{s - d_{ij}^{ax}}{\tau_s}\right) \right] \mathcal{H}(s - d_{ij}^{ax})$$

External Current

# Model Advantages

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- Hodgkin-Huxley
  - Accurate Modeling
  - Predicts membrane potentials due to pharmacological blocking of ion channels
- Integrate & Fire
  - Easy implementation
  - Computation-light
- Spike Response Model
  - Includes refractory phase

# Rate Coding v. Temporal Coding

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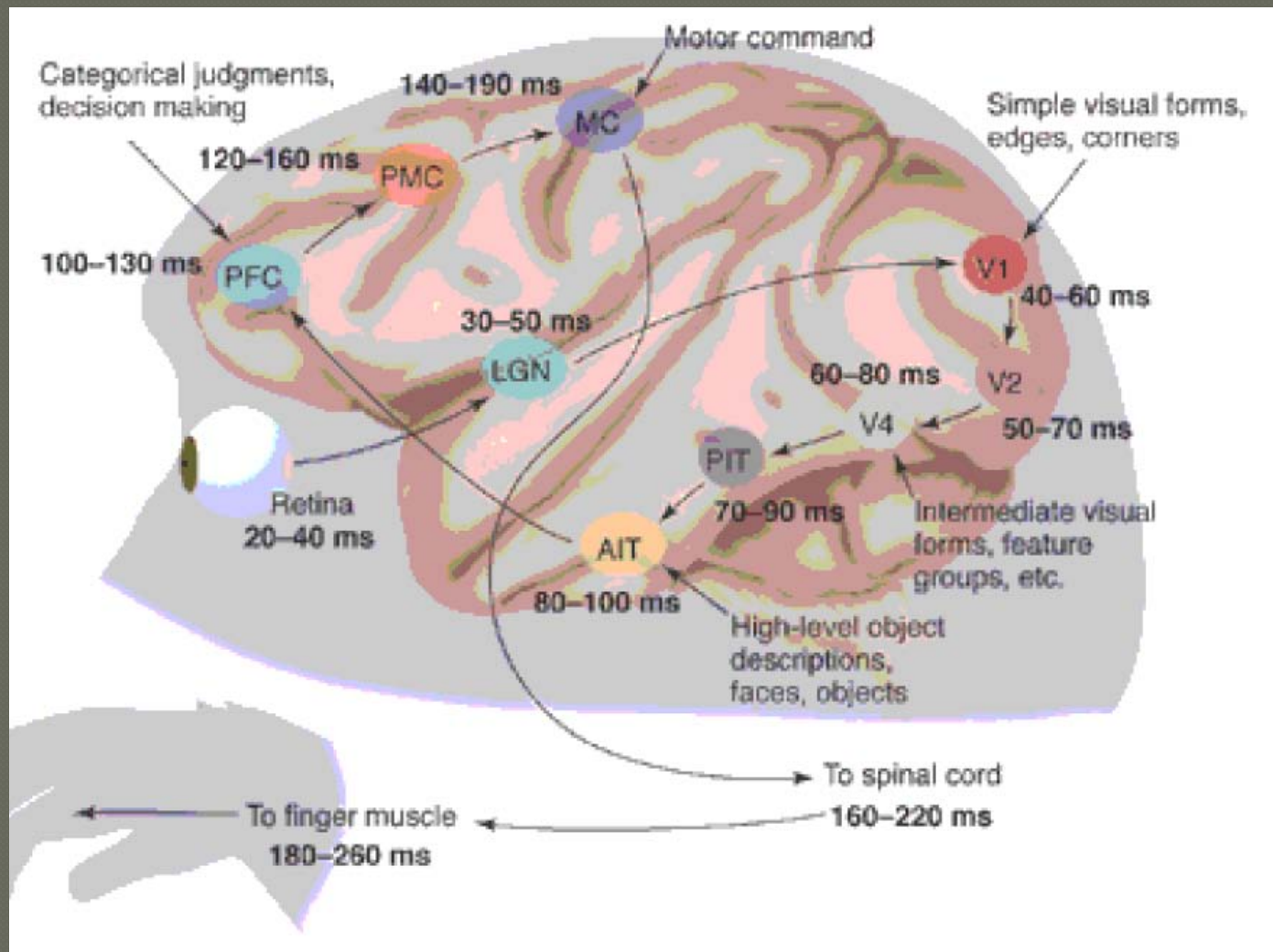
## ◉ Rate Coding

- Information transmitted by rates
- I.E. number of spikes per unit time

## ◉ Temporal Coding

- The exact timing of spikes matter

# Rate Coding v. Temporal Coding



# (Trad.) Temporal Computing

$$u_j(t) = \sum_{t_j^{(f)} \in \mathcal{F}_j} \eta_j(t - t_j^{(f)}) + \sum_{i \in \Gamma_j} \sum_{t_i^{(f)} \in \mathcal{F}_i} w_{ij} \epsilon_{ij}(t - t_i^{(f)})$$

$$\sum_{i \in \Gamma_j} w_{ij} \epsilon_{ij}(t_j - t_i) = \sum_{i \in \Gamma_j} w_{ij} \lambda_{ij}(t_j - t_i - \Delta_{ij}) = \vartheta$$

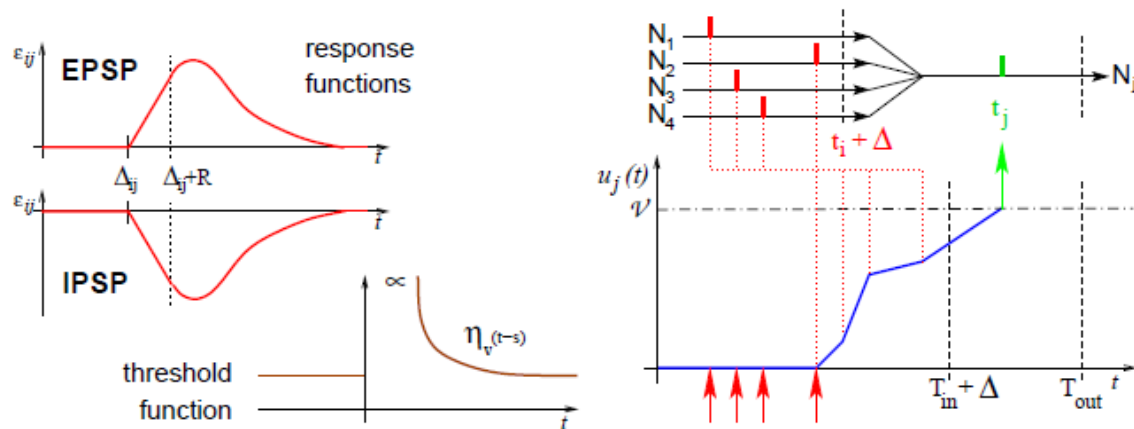


Figure 9: Shapes of postsynaptic potential (EPSP or IPSP) for computing a weighted sum in temporal coding. Right: Example variation of neuron  $N_j$  membrane potential for computing  $\sum_{i \in \Gamma_j} \alpha_{ij} x_i$  and resulting firing time  $t_j$ . All the delays  $\Delta_{ij}$  have been set equal to  $\Delta$ . Neuron  $N_4$  (third firing) is inhibitory whereas the other three are excitatory. The slopes of the PSPs are modulated by the synaptic efficacies  $w_{ij}$ .

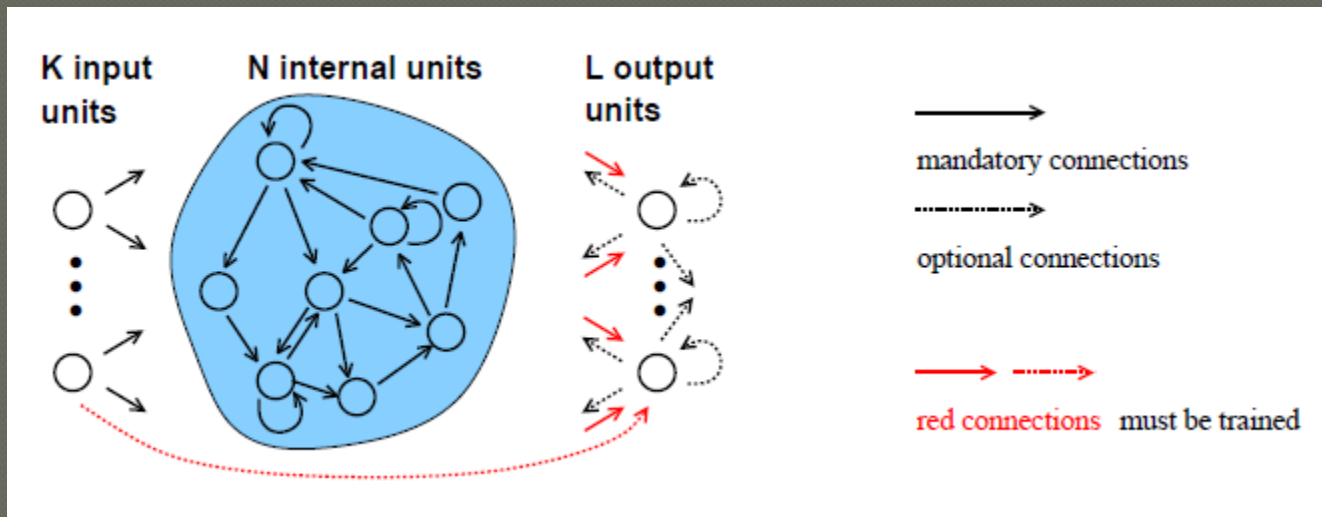
# Network Topology and Dynamics

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- Reviewed models describe single neurons, still need to create networks
- Traditional Architectures
  - Use temporal coding to reduce SNN to NN
  - Refer to previous slide
- Echo State Networks & Liquid State Machines

# Echo State Networks

- Produce an echo state network
- Sample network training dynamics
- Compute output weights, use any linear regression algorithm
- SNs implemented in ESN outperform traditional ESNs





# Liquid State Machines

- Turns time varying input into a spatiotemporal pattern of activation
- Large number of non-linear activation states
- Activations go into readout neuron(s) (linear discriminate units)

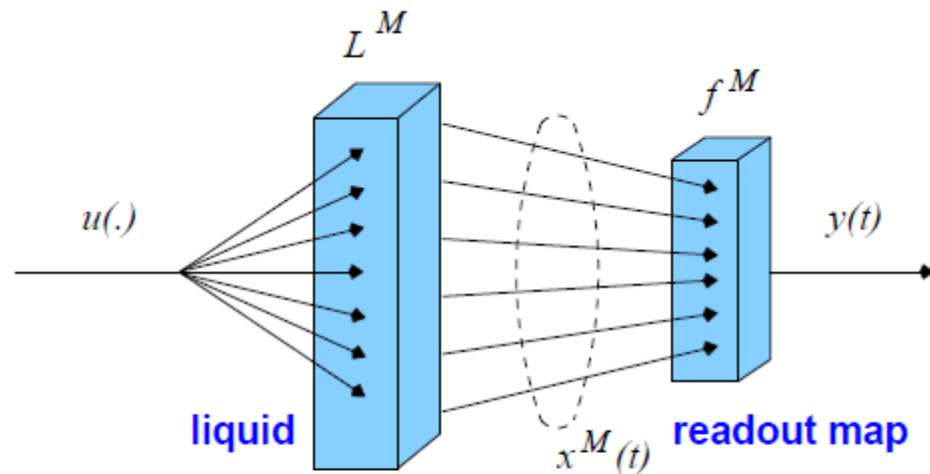


Figure 11: Architecture of a “Liquid State Machine”. A continuous stream of values  $u(\cdot)$  is injected as input into the liquid filter  $L^M$ . A sufficiently complex excitable “liquid medium” creates, at time  $t$ , the *liquid state*  $x^M(t)$ , which is transformed by a memoryless *readout map*  $f^M$  to generate output  $y(t)$ .

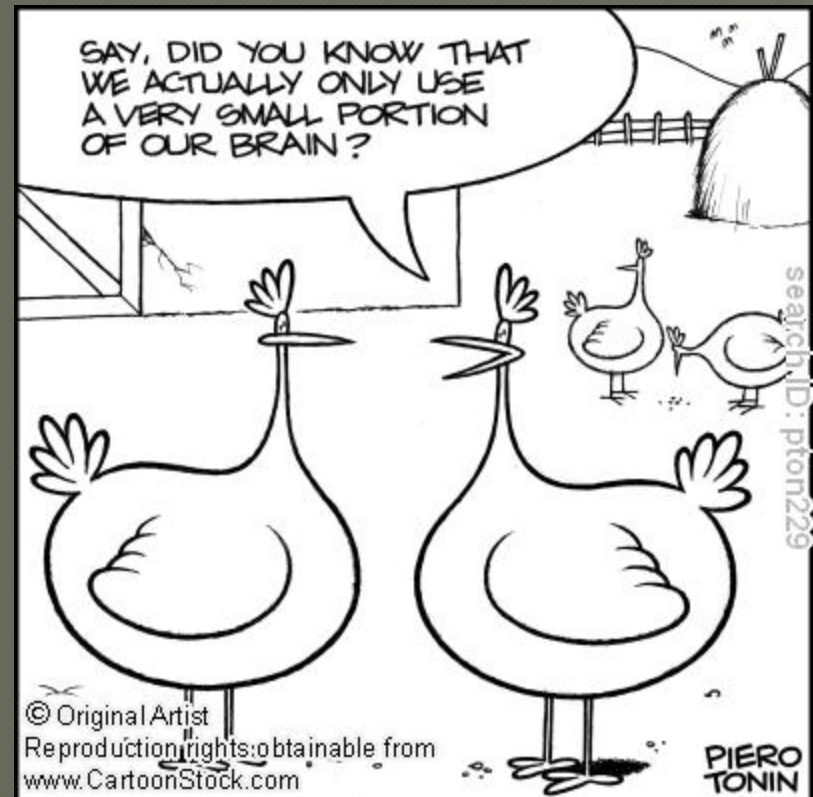
# Cell Assemblies

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- “A group of neurons with strong mutual excitatory connections.”
- Excite one, excite all (many)
- “Grandmother Neural Groups”
- Synfire chain: pool of in-sync neurons
- Transient synchrony
  - Leads to collective sync. event; computational building block, “many variables are cur. ~equal”
- Polychronization
  - “reproducible time-locked but not synchronous firing patterns”

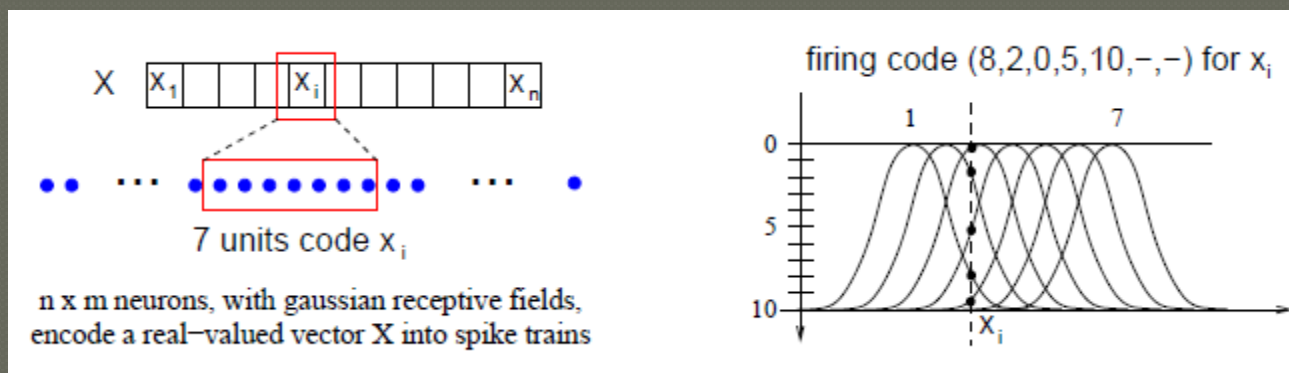
# Learning Rules

- Traditional Methods
- New SNN Methods



# Applying Traditional Learning Rules to SNN

- Hopfield Networks (Maass & Natschlager)
- Kohonen SOMs (Ruf & Schmitt)
- RBF Networks (Natschlager & Rug)
- ML RBF Networks (Bohte, La Poutre & Kok)
- SNN shown to be universal function approximators

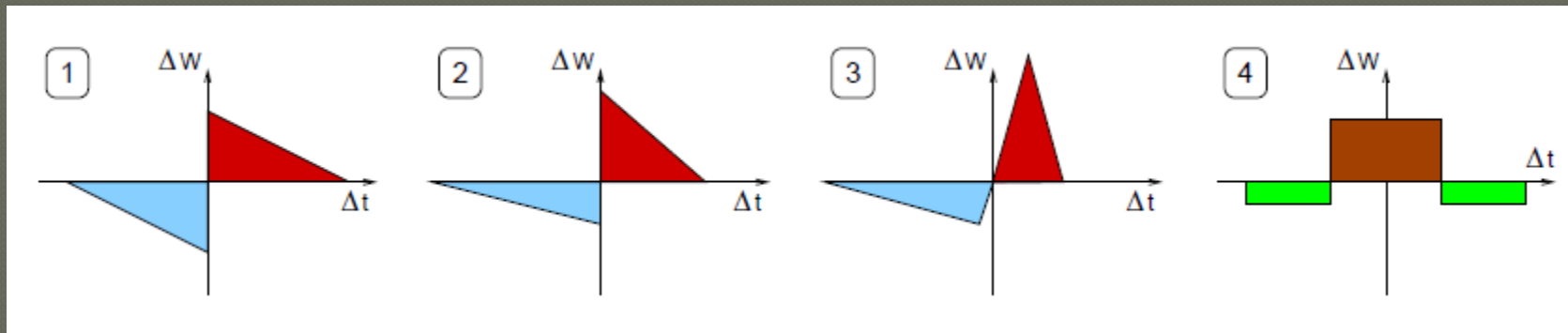
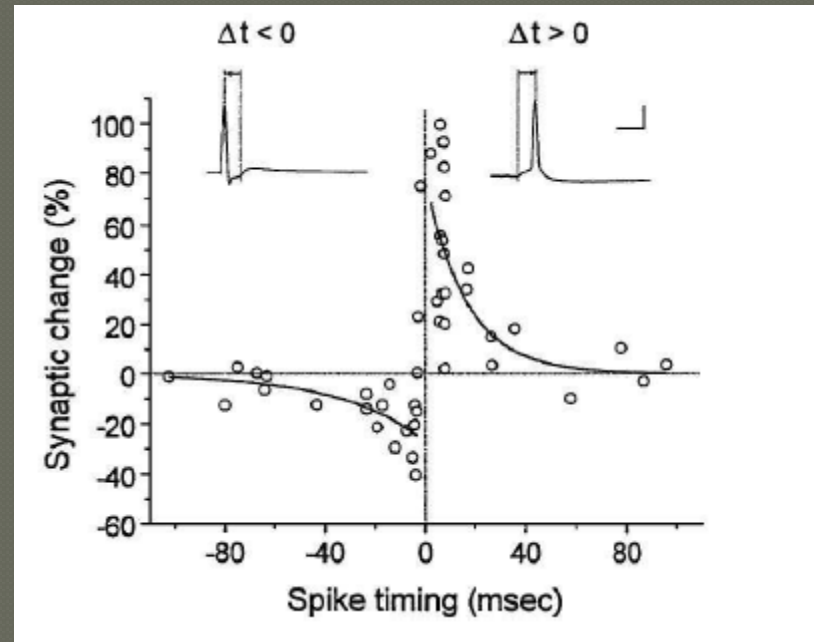


# Hebbian-based Learning

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- “When a pre-synaptic neuron repeatedly fires right before a post-synaptic neuron fires, the weight between the two neurons increases.”
- Hebbian Properties
  - Synaptic Scaling
  - Synaptic Redistribution
  - Spike-timing dependent synaptic plasticity

# Spike-timing Dependent Synaptic Plasticity (STDP)



# SNN Learning Theory Models

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- Maximization of mutual information
- BCM model
- Minimization of entropy
  - Minimize the response variability in the post-synaptic neuron given a particular input pattern

# Software & Hardware Implementation

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## ⊙ Event-driven Simulation

- Vs. time-driven simulation
- Most of the time neurons aren't firing, so
- Calculate when firing events occur, not what every neuron is doing at every time step
- Delayed firing problem

## ⊙ Parallel

- SpikeNET
- DAMNED simulator



# Applications

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- Hopfield and Brody, Digit Recognition
  - Generalize from small number of examples
  - Robust to noise
  - Uses temporal integration of transient synchrony
  - Time warp invariant
  - A set of neurons fire synchronously to a particular input (transient synchrony)
- Many examples in
  - Speech processing
  - Computer Vision

# Discussion

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## ○ Spiking Neuron Networks

- Biologically motivated
- Computationally difficult without simplification
- Traditional learning rules don't take advantage of timing sequencing
- New learning rules will have to be forthcoming before SNN show their potential